Antenna Placement of Samsung Galaxy SII

Antenna Placement of HTC Desire S

# Lect9

# Microstrip antenna

Antenna Placement of iPhone 5

# Rectangular Microstrip Antenna

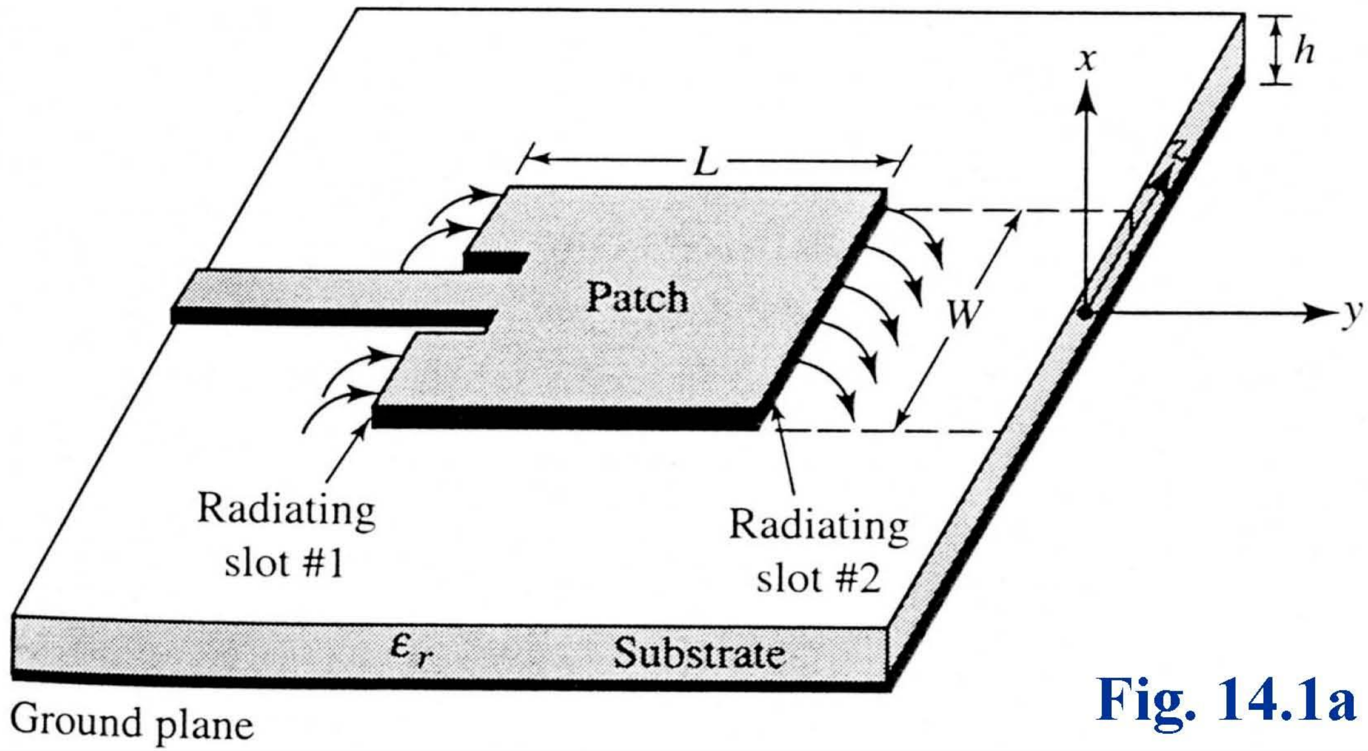



Fig. 14.1a

# Side View

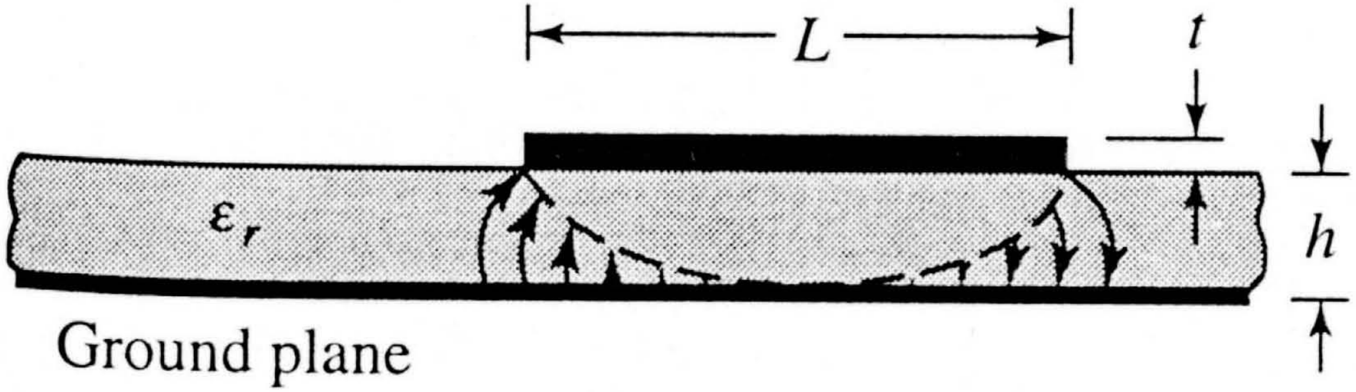


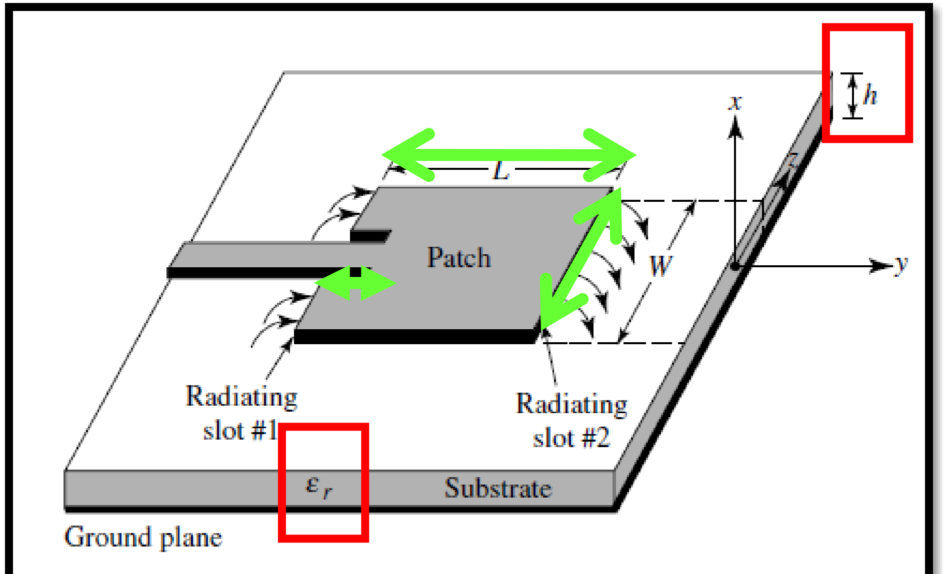

**Fig. 14.1b**

# Methods of analysis

- Transmission line model
- Cavity model
- Full wave model (most complex but accurate)

## Rectangular Patch

### Fringing fields are responsible of radiation



(a) Microstrip antenna

- Analyzed using transmission line model and cavity model.

Rectangular Microstrip antenna can be represented as an **array** of two radiating narrow slot each of width ***W*** and height ***h*** separated by T.L of a distance ***L.***

This array has a broadside radiation pattern (the peak radiation is in the +x-direction).

# Fringing Effect

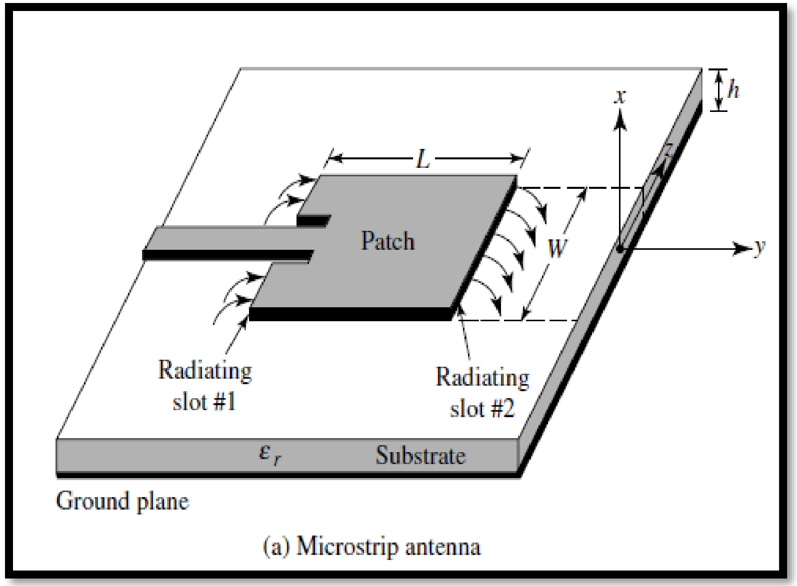
Due to finite length of patch electric field at edge undergo fringing.

**Fringing effect makes**

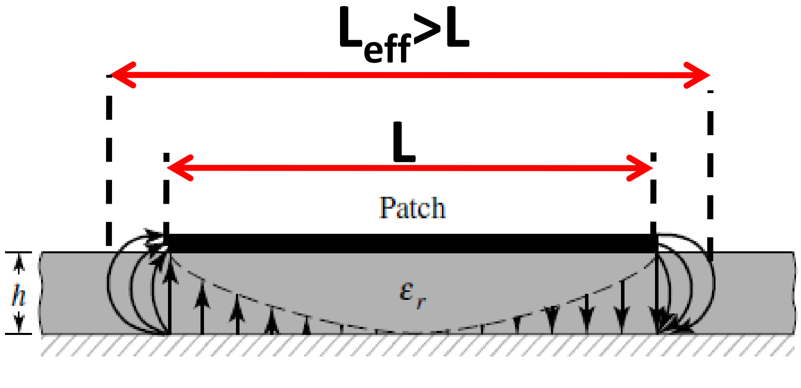
- the effective electrical length of the patch looks longer than its physical length .   $L_{eff}=L+2\Delta L$
- Effective dielectric constant $\varepsilon_{reff} < \varepsilon_r$ since electric lines travels in air and in substrate(dielectric)

***The amount of fringing depend on***

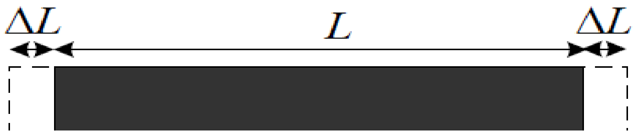
- L/h if L/h>>1 fringing is reduced
- Dielectric material as $\varepsilon_r$>>1 fringing is reduced.



(a) Microstrip antenna



(b) Side view



# Design Equations:

$$W=\frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}}$$

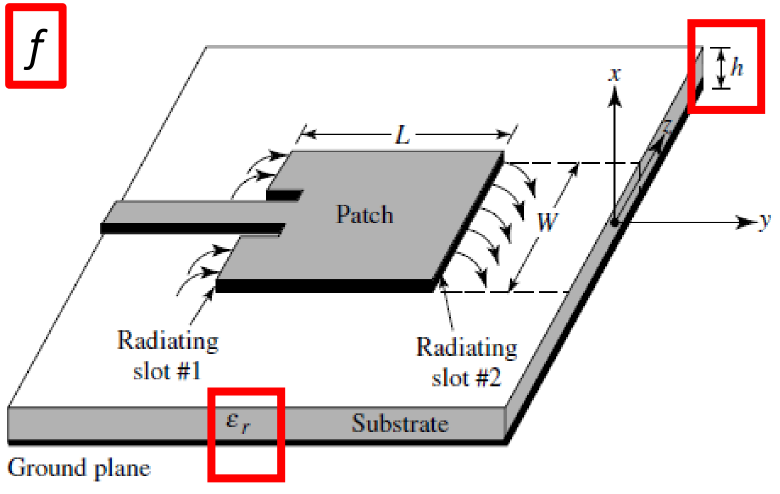
$$\varepsilon_{reff}=\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}[1+\frac{12h}{W}]^{-1/2}$$

$$\frac{\Delta L}{h}=0.412\frac{(\varepsilon_{reff}+0.3)(\frac{W}{h}+0.264)}{(\varepsilon_{reff}-0.258)(\frac{W}{h}+0.8)}$$

$$L=\frac{c}{2f_r\sqrt{\varepsilon_{reff}}}-2\Delta L$$

---

$$(f_r)_{010}=\frac{c}{2\sqrt{\varepsilon_{reff}}L_{eff}}$$



(a) Microstrip antenna

**the length of a *half-wave patch is slightly less than* a half wavelength in the dielectric substrate material(.49λg) ; The amount of length reduction depends on $\varepsilon_r$ ,*h, and W. Formulas are available to estimate the resonant length with* empirical adjustments**.

### Example 14.1

Design a rectangular microstrip antenna using a substrate (RT/duroid 5880) with dielectric constant of 2.2, $h = 0.1588$ cm (0.0625 inches) so as to resonate at 10 GHz.

*Solution*: Using (14-6), the width $W$ of the patch is

$$W = \frac{30}{2(10)}\sqrt{\frac{2}{2.2+1}} = 1.186 \text{ cm } (0.467 \text{ in})$$

The effective dielectric constant of the patch is found using (14-1), or

$$\epsilon_{\text{reff}} = \frac{2.2+1}{2} + \frac{2.2-1}{2}\left(1 + 12\frac{0.1588}{1.186}\right)^{-1/2} = 1.972$$

The extended incremental length of the patch $\Delta L$ is, using (14-2)

$$\Delta L = 0.1588(0.412)\frac{(1.972+0.3)\left(\frac{1.186}{0.1588}+0.264\right)}{(1.972-0.258)\left(\frac{1.186}{0.1588}+0.8\right)}$$
$$= 0.081 \text{ cm } (0.032 \text{ in})$$

The actual length $L$ of the patch is found using (14-3), or

$$L = \frac{\lambda}{2} - 2\Delta L = \frac{30}{2(10)\sqrt{1.972}} - 2(0.081) = 0.906 \text{ cm } (0.357 \text{ in})$$

Finally the effective length is

$$L_e = L + 2\Delta L = \frac{\lambda}{2} = 1.068 \text{ cm } (0.421 \text{ in})$$

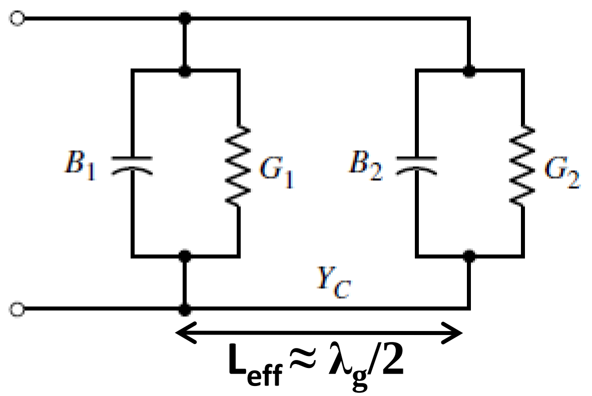
• Inset feed design:

Each radiating slot is represented by a parallel equivalent admittance $Y$

$$Y_1 = G_1 + jB_1$$

Since slot #2 is identical to slot #1, its equivalent admittance is

$$Y_2 = Y_1, \quad G_2 = G_1, \quad B_2 = B_1$$



(b) Transmission model equivalent

At resonance input impedance is purely resistive

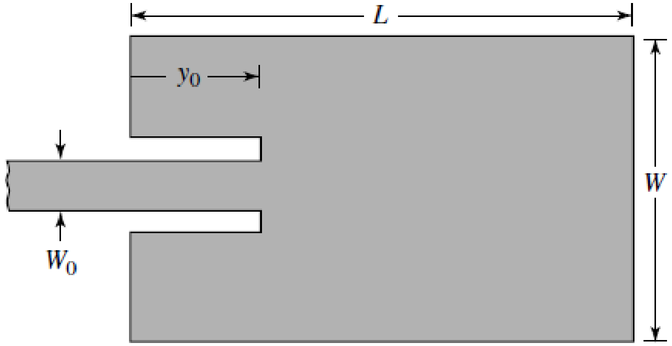
Reactance cancelled each other . Hence for $TM_{010}$

$$R_{in} = \frac{1}{2(G_1 \pm G_{12})}$$

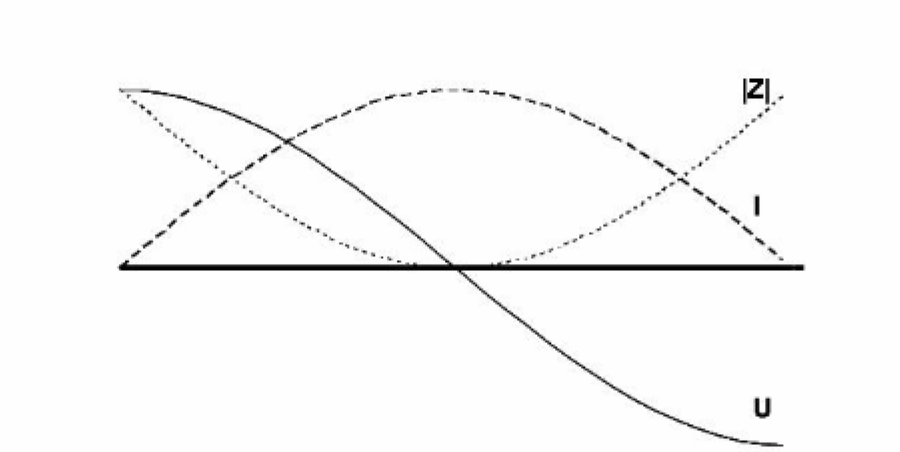
Where:

Rin input impedance at edge of patch(i.e at y=0)

G1 conductance of slot 1 ,G12 mutual conductance between slot1 and slot2.(+) for odd distribution of resonant voltage (as in $TM_{010}$).




(a) Recessed microstrip-line feed




Voltage (U), current (I) and impedance (|Z|) distribution along the patch's resonant length

$$R_{in}(y = y_0) = \frac{1}{2(G_1 \pm G_{12})}\cos^2\left(\frac{\pi}{L}y_0\right)$$

It is desired to have $R_{in}(y=y_0) = 50\Omega$

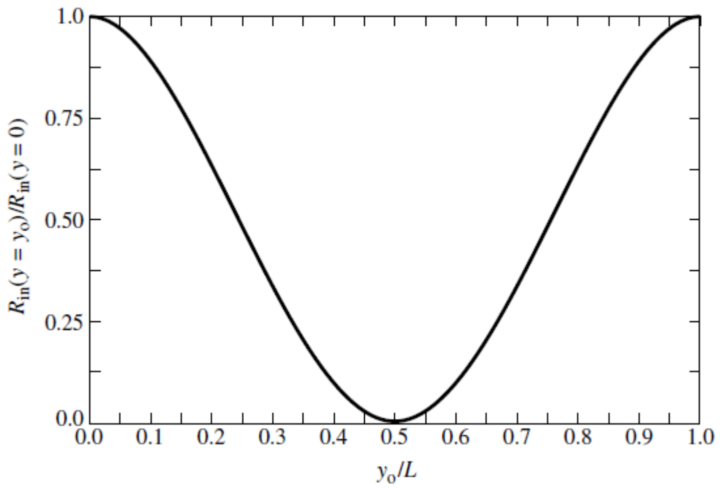
$$G_1 = \frac{I}{120\pi^2},$$

where

$$I = \int_0^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 \sin^3\theta d\theta = -2 + \cos X + X \cdot S_i(X) + \frac{\sin X}{X},$$

and $X = k_0 W$, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. $S_i$ denotes the sine integral, $\int_0^x (\sin y)/y dy$.

$$Si(x) = x - \frac{x^3}{3*3!} + \frac{x^5}{5*5!} - \frac{x^7}{7*7!} + \text{.........} =$$




(b) Normalized input resistance

$$R_{in} = \frac{1}{2(G_1 + G_{12})}$$

$$G_1 = \frac{1}{120\pi^2} \int_0^{\pi} \left[ \frac{\sin(\frac{WK_0}{2}\cos\theta)}{\cos\theta} \right]^2 \sin^3\theta d\theta$$

$$G_{12} = \frac{1}{120\pi^2} \int_0^{\pi} \left[ \frac{\sin(\frac{WK_0}{2}\cos\theta)}{\cos\theta} \right]^2 J_0(K_0 L \sin\theta) \sin^3\theta d\theta$$

$$y_0 = \frac{L}{\pi} \cos^{-1} \sqrt{\frac{50}{R_{in}}}$$

## Example 14.2

A microstrip antenna with overall dimensions of $L = 0.906$ cm (0.357 inches) and $W = 1.186$ cm (0.467 inches), substrate with height $h = 0.1588$ cm (0.0625 inches) and dielectric constant of $\epsilon_r = 2.2$, is operating at 10 GHz. Find:

**a.** The input impedance.

**b.** The position of the inset feed point where the input impedance is 50 ohms.

*Solution*:

$$\lambda_0 = \frac{30}{10} = 3 \text{ cm}$$

Using (14-12) and (14-12a)

$$G_1 = 0.00157 \text{ siemens}$$

which compares with $G_1 = 0.00328$ using (14-8a). Using (14-18a)

$$G_{12} = 6.1683 \times 10^{-4}$$

Using (14-17) with the (+) sign because of the odd field distribution between the radiating slots for the dominant $TM_{010}$ mode

$$R_{in} = 228.3508 \text{ ohms.}$$

Since the input impedance at the leading radiating edge of the patch is 228.3508 ohms while the desired impedance is 50 ohms, the inset feed point distance $y_0$ is obtained using (14-20a). Thus

$$50 = 228.3508 \cos^2\left(\frac{\pi}{L} y_0\right)$$

or

$$y_0 = 0.3126 \text{ cm (0.123 inches)}$$

# Cavity Model for Microstrip Antenna

- The normalized fields within the dielectric substrate (between the patch and the ground plane) can be found more accurately by treating that region as a cavity bounded by electric conductors (above and below it) and by magnetic walls (to simulate an open circuit) along the perimeter of the patch.

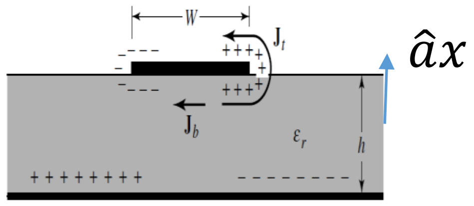



**Figure 14.12** Charge distribution and current density creation on microstrip patch.

Small h/W approximately, the current flow to the top would be zero( open circuit), which ideally would not create any tangential magnetic field components to the edges of the patch. This would allow the four side

- walls to be modeled as perfect magnetic. Also the material within the cavity assumed to be lossless, i.e. cavity would not radiate and its input impedance would be purely reactive.

- Later radiation will be considered through the material effective loss tangent δeff. The effective loss tangent is chosen appropriately to represent the loss mechanism of the cavity, which now behaves as an antenna and is taken as the reciprocal of the antenna quality factor Q (δeff = 1/Q)
- height of the substrate is very small ($h \ll \lambda$) electric field is nearly normal to the surface of the patch. Therefore only $TM^x$ field will be considered within the cavity.(no H in direction of propagation)

- The vector potential *Ax* must satisfy the homogeneous wave equation of

$$\nabla^2 A_x + k^2 A_x = 0$$

*A → magnetic vector potential*
*At same direction of E*

whose solution is written in general, using the separation of variables,

$$A_x = [A_1 \cos(k_x x) + B_1 \sin(k_x x)][A_2 \cos(k_y y) + B_2 \sin(k_y y)] \cdot [A_3 \cos(k_z z) + B_3 \sin(k_z z)] \qquad (14\text{-}22)$$

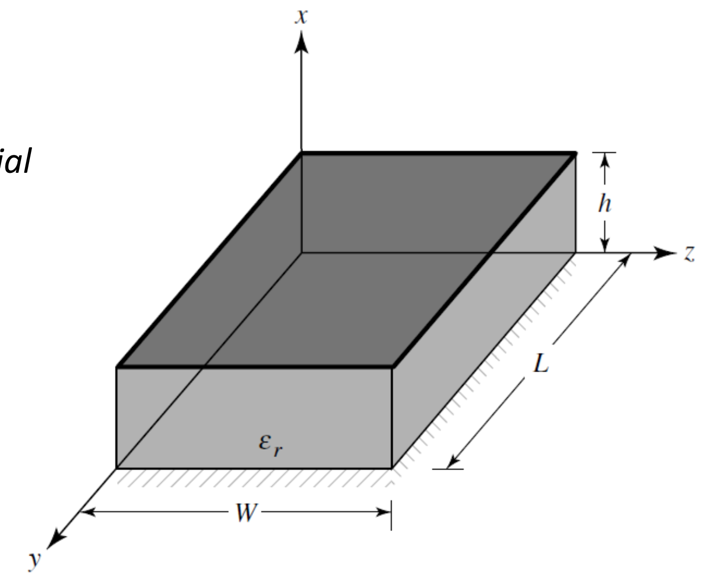



**Figure 14.13** Rectangular microstrip patch geometry.

The electric and magnetic fields within the cavity are related to the vector potential $A_x$ by

$$E_x = -j\frac{1}{\omega\mu\epsilon}\left(\frac{\partial^2}{\partial x^2} + k^2\right)A_x \qquad H_x = 0$$

$$E_y = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 A_x}{\partial x \partial y} \qquad H_y = \frac{1}{\mu}\frac{\partial A_x}{\partial z}$$

$$E_z = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 A_x}{\partial x \partial z} \qquad H_z = -\frac{1}{\mu}\frac{\partial A_x}{\partial y}$$

subject to the boundary conditions of

$$E_y(x' = 0, 0 \le y' \le L, 0 \le z' \le W) = E_y(x' = h, 0 \le y' \le L, 0 \le z' \le W) = 0$$

$$H_y(0 \le x' \le h, 0 \le y' \le L, z' = 0) = H_y(0 \le x' \le h, 0 \le y' \le L, z' = W) = 0$$

$$H_z(0 \le x' \le h, y' = 0, 0 \le z' \le W) = H_z(0 \le x' \le h, y' = L, 0 \le z' \le W) = 0$$

Applying the boundary conditions

$$E_y(x' = 0, 0 \le y' \le L, 0 \le z' \le W) = 0$$
$$E_y(x' = h, 0 \le y' \le L, 0 \le z' \le W) = 0,$$

$\rightarrow$ $B_1 = 0$ and

$$k_x = \frac{m\pi}{h}, \quad m = 0, 1, 2, \ldots$$

$$H_y(0 \le x' \le h, 0 \le y' \le L, z' = 0) = 0$$
$$H_y(0 \le x' \le h, 0 \le y' \le L, z' = W) = 0,$$

$\rightarrow$ $B_3 = 0$ and

$$k_z = \frac{p\pi}{W}, \quad p = 0, 1, 2, \ldots$$

$$H_z(0 \le x' \le h, y' = 0, 0 \le z' \le W) = 0$$
$$H_z(0 \le x' \le h, y' = L, 0 \le z' \le W) = 0,$$

$\rightarrow$ $B_2 = 0$ and

$$k_y = \frac{n\pi}{L}, \quad n = 0, 1, 2, \ldots.$$

*where m, n, p represent, respectively, the number of half-cycle field variations along the x, y, z directions.*

the final form for the vector potential $A_x$ within the cavity is

$$A_x = A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z')$$

$$\because\ k_x^2 + k_y^2 + k_z^2 = \left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{W}\right)^2 = k_r^2 = \omega_r^2 \mu\epsilon$$

the resonant frequencies for the cavity are given by

$$(f_r)_{mnp} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{h}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{W}\right)^2}$$

$$A_x = A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z')$$

The electric and magnetic fields within the cavity are related to the vector potential $A_x$ by

$$E_x = -j\frac{1}{\omega\mu\epsilon}\left(\frac{\partial^2}{\partial x^2} + k^2\right) A_x \qquad H_x = 0$$

$$E_y = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 A_x}{\partial x \partial y} \qquad H_y = \frac{1}{\mu}\frac{\partial A_x}{\partial z}$$

$$E_z = -j\frac{1}{\omega\mu\epsilon}\frac{\partial^2 A_x}{\partial x \partial z} \qquad H_z = -\frac{1}{\mu}\frac{\partial A_x}{\partial y}$$

(14-23)

$$E_x = -j\frac{(k^2 - k_x^2)}{\omega\mu\epsilon} A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z')$$

$$E_y = -j\frac{k_x k_y}{\omega\mu\epsilon} A_{mnp} \sin(k_x x') \sin(k_y y') \cos(k_z z')$$

$$E_z = -j\frac{k_x k_z}{\omega\mu\epsilon} A_{mnp} \sin(k_x x') \cos(k_y y') \sin(k_z z')$$

$$H_x = 0$$

$$H_y = -\frac{k_z}{\mu} A_{mnp} \cos(k_x x') \cos(k_y y') \sin(k_z z')$$

$$H_z = \frac{k_y}{\mu} A_{mnp} \cos(k_x x') \sin(k_y y') \cos(k_z z')$$

If $L > W > h$, the mode with the lowest frequency (dominant mode) is the $TM^x_{010}$ whose resonant frequency is given by

$$(f_r)_{010} = \frac{1}{2L\sqrt{\mu\epsilon}} = \frac{\upsilon_0}{2L\sqrt{\epsilon_r}}$$

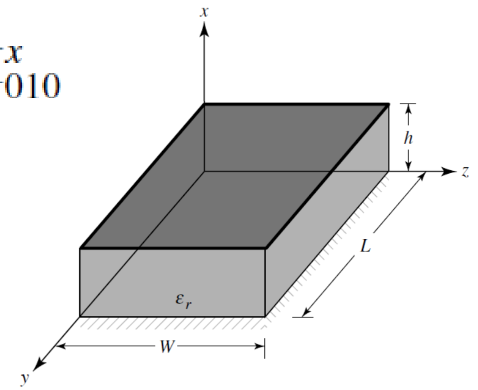



**Figure 14.13** Rectangular microstrip patch geometry.

Higher order mode can also be exists

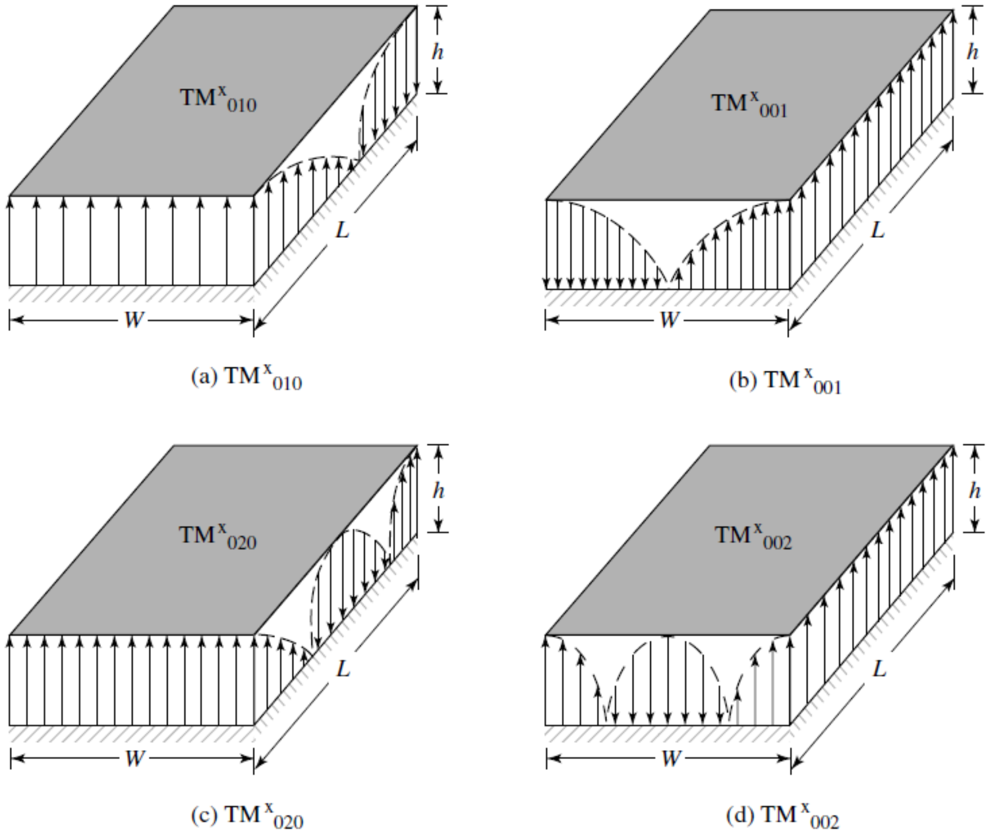



**Figure 14.14** Field configurations (modes) for rectangular microstrip patch.

Assuming that the dominant mode within the cavity is the $TM^x_{010}$

$$E_x = -j\frac{(k^2 - k_x^2)}{\omega\mu\epsilon} A_{mnp} \cos(k_x x') \cos(k_y y') \cos(k_z z')$$

$$E_y = -j\frac{k_x k_y}{\omega\mu\epsilon} A_{mnp} \sin(k_x x') \sin(k_y y') \cos(k_z z')$$

$$E_z = -j\frac{k_x k_z}{\omega\mu\epsilon} A_{mnp} \sin(k_x x') \cos(k_y y') \sin(k_z z')$$

$$H_x = 0$$

$$H_y = -\frac{k_z}{\mu} A_{mnp} \cos(k_x x') \cos(k_y y') \sin(k_z z')$$

$$H_z = \frac{k_y}{\mu} A_{mnp} \cos(k_x x') \sin(k_y y') \cos(k_z z')$$

→

$$E_x = E_0 \cos\left(\frac{\pi}{L} y'\right)$$

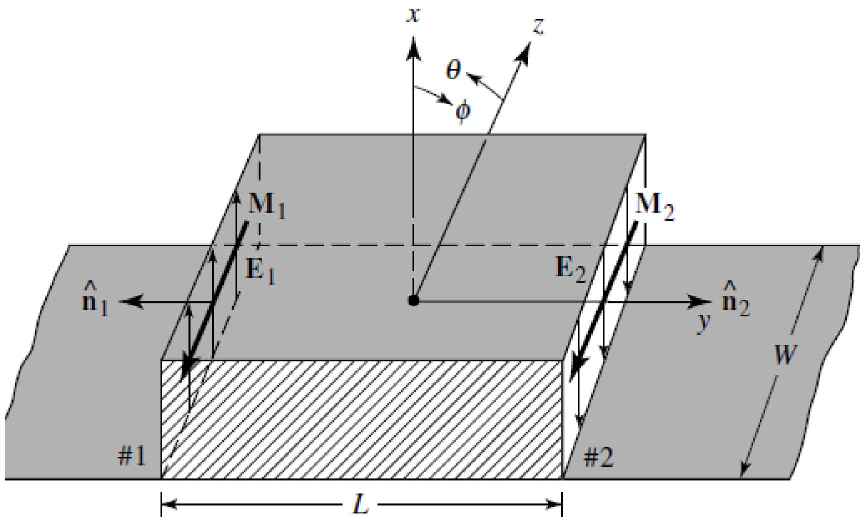
$$H_z = H_0 \sin\left(\frac{\pi}{L} y'\right)$$

$$E_y = E_z = H_x = H_y = 0$$

where $E_0 = -j\omega A_{010}$

$H_0 = (\pi/\mu L) A_{010}$.

and $A_{mnp}$ represents the amplitude coefficients of each $mnp$ mode auxiliary potential functions **A** and **F** generated, respectively, by **J** and **M** are found $\mathbf{M}_s = -2\hat{\mathbf{n}} \times \mathbf{E}_a$




$$\mathbf{F} = \frac{\epsilon}{4\pi} \iint_S \mathbf{M}_s \frac{e^{-jkR}}{R} ds' \simeq \frac{\epsilon e^{-jkr}}{4\pi r} \mathbf{L}$$

$$\mathbf{L} = \int \mathbf{M}_s e^{jkr' \cos\psi} ds'$$

**J** at the bottom of the patch set to zero.
Also it was argued that the tangential magnetic fields along the edges of the patch are very small, so **J**s will be zero so only **M**s *is exist as a source.*
*Due to large ground and image theory*

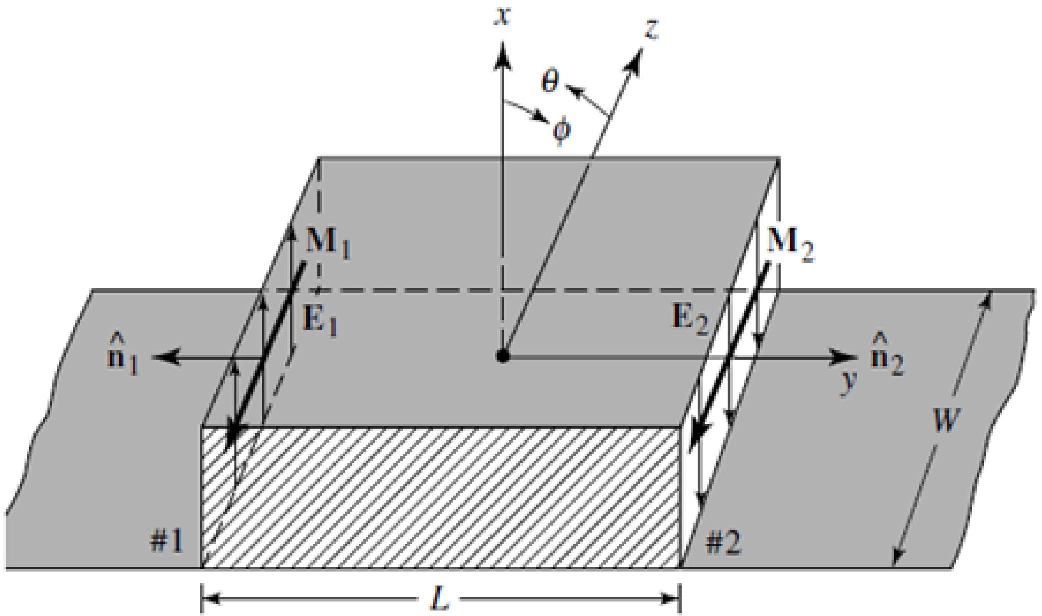
$$\mathbf{M}_s = -2\hat{\mathbf{n}} \times \mathbf{E}_a$$

**TABLE 3.1 Dual Equations for Electric (J) and Magnetic (M) Current Sources**

| Electric Sources ($\mathbf{J} \neq 0, \mathbf{M} = 0$) | Magnetic Sources ($\mathbf{J} = 0, \mathbf{M} \neq 0$) |
|---|---|
| $\nabla \times \mathbf{E}_A = -j\omega\mu\mathbf{H}_A$ | $\nabla \times \mathbf{H}_F = j\omega\epsilon\mathbf{E}_F$ |
| $\nabla \times \mathbf{H}_A = \mathbf{J} + j\omega\epsilon\mathbf{E}_A$ | $-\nabla \times \mathbf{E}_F = \mathbf{M} + j\omega\mu\mathbf{H}_F$ |
| $\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}$ | $\nabla^2\mathbf{F} + k^2\mathbf{F} = -\epsilon\mathbf{M}$ |
| $\mathbf{A} = \frac{\mu}{4\pi}\iiint_V \mathbf{J}\frac{e^{-jkR}}{R}dv'$ | $\mathbf{F} = \frac{\epsilon}{4\pi}\iiint_V \mathbf{M}\frac{e^{-jkR}}{R}dv'$ |
| $\mathbf{H}_A = \frac{1}{\mu}\nabla \times \mathbf{A}$ | $\mathbf{E}_F = -\frac{1}{\epsilon}\nabla \times \mathbf{F}$ |
| $\mathbf{E}_A = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{A})$ | $\mathbf{H}_F = -j\omega\mathbf{F} - j\frac{1}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{F})$ |

H same direction of M
E perp. direction of M

so if Ltheta prod Ephi
and if Lphi prod Etheta



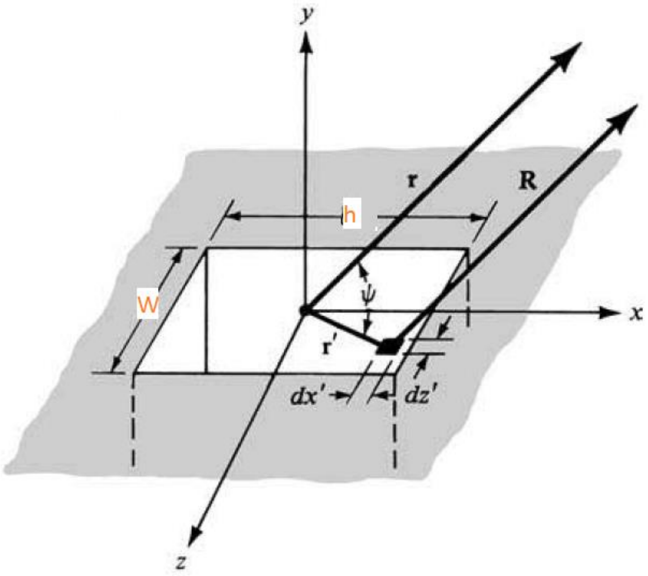

$$\mathbf{F} = \frac{\epsilon}{4\pi} \iint_S \mathbf{M}_s \frac{e^{-jkR}}{R} ds'$$

$$\mathbf{H}_F = -j\omega\mathbf{F}$$

$M_z$ --> $M_\theta$ --> $F_\theta$ $H_\theta$ $E_\phi$

$$E_\phi = \eta\, H_\theta$$

$$E_\phi \simeq -\frac{jke^{-jkr}}{4\pi r} \iint_S M_z \sin\theta \; e^{+jkr'\cos\psi}\, ds'$$




$$r'\cos\psi = \mathbf{r}'\cdot\hat{\mathbf{a}}_r = (\hat{\mathbf{a}}_x x' + \hat{\mathbf{a}}_z z')\cdot(\hat{\mathbf{a}}_x \sin\theta\cos\phi + \hat{\mathbf{a}}_y \sin\theta\sin\phi + \hat{\mathbf{a}}_z\cos\theta)$$

$$= x'\sin\theta\cos\phi + z'\cos\theta \qquad \text{(12-15b)}$$

$$ds' = dx'\,dz'$$

# Radiating slots

$$E_r \simeq E_\theta \simeq 0$$

$$E_\phi \simeq -\frac{jke^{-jkr}}{4\pi r} \int_{-W/2}^{W/2}\int_{-h/2}^{h/2} -2E_0 \cos\left(\frac{\pi}{L}y'\right)\Big|_{y'=0 \text{ or } y'=L} \sin\theta \; e^{+jk(x'\sin\theta\cos\phi + z'\cos\theta)} \, dx'\, dz'$$

at slots

note axis at slots

$$\int_{-c/2}^{+c/2} e^{j\alpha z}\, dz = c\left[\frac{\sin\left(\frac{\alpha}{2}c\right)}{\frac{\alpha}{2}c}\right]$$

$$E_\phi = +j\frac{k_0 h W E_0 e^{-jk_0 r}}{2\pi r}\left\{\sin\theta \frac{\sin(X)}{X}\frac{\sin(Z)}{Z}\right\}$$

$$X = \frac{k_0 h}{2}\sin\theta\cos\phi$$

$$Z = \frac{k_0 W}{2}\cos\theta$$

the array factor for the two elements, of the same magnitude and phase, separated by a distance $L_e$ along the $y$ direction is

$$(AF)_y = 2\cos\left(\frac{k_0 L_e}{2}\sin\theta\sin\phi\right)$$

the total electric field for the two slots (also for the microstrip antenna) is

$$E_\phi^t = +j\frac{k_0 h W E_0 e^{-jk_0 r}}{\pi r}\left\{\sin\theta\frac{\sin(X)}{X}\frac{\sin(Z)}{Z}\right\} \times \cos\left(\frac{k_0 L_e}{2}\sin\theta\sin\phi\right) \qquad (14\text{-}43)$$

where

$$X = \frac{k_0 h}{2}\sin\theta\cos\phi \qquad (14\text{-}43a)$$

$$Z = \frac{k_0 W}{2}\cos\theta \qquad (14\text{-}43b)$$

$$E_\phi^t = +j\frac{k_0 h W E_0 e^{-jk_0 r}}{\pi r}\left\{\sin\theta \frac{\sin(X)}{X}\frac{\sin(Z)}{Z}\right\} \times \cos\left(\frac{k_0 L_e}{2}\sin\theta\sin\phi\right) \qquad X = \frac{k_0 h}{2}\sin\theta\cos\phi \qquad Z = \frac{k_0 W}{2}\cos\theta$$

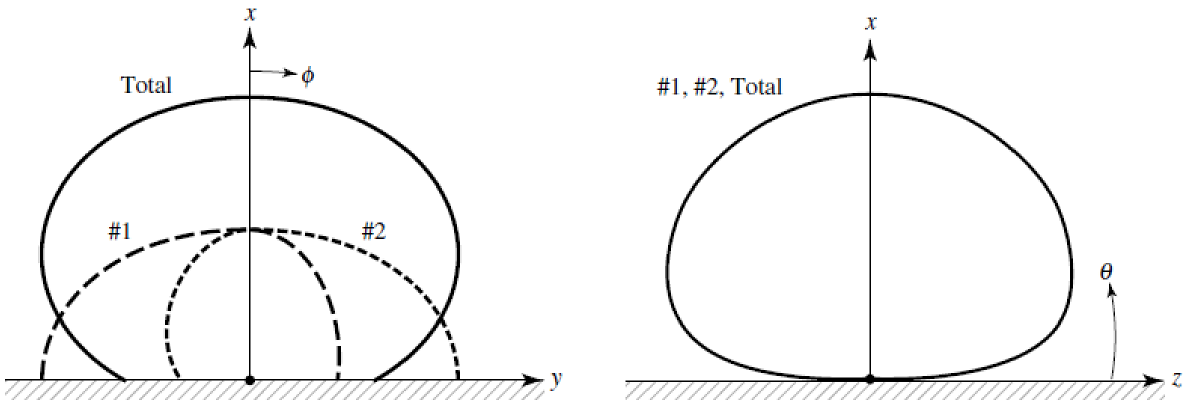
**$E$-Plane** ($\theta = 90°, 0° \le \phi \le 90°$ **and** $270° \le \phi \le 360°$)
$x$-$y$ plane

$$E_\phi^t = +j\frac{k_0 W V_0 e^{-jk_0 r}}{\pi r}\left\{\frac{\sin\left(\frac{k_0 h}{2}\cos\phi\right)}{\frac{k_0 h}{2}\cos\phi}\right\}\cos\left(\frac{k_0 L_e}{2}\sin\phi\right)$$

where $V_0 = hE_0$

**$H$-Plane** ($\phi = 0°, 0° \le \theta \le 180°$)
$x$-$z$ plane

$$E_\phi^t \simeq +j\frac{k_0 W V_0 e^{-jk_0 r}}{\pi r}\left\{\sin\theta\frac{\sin\left(\frac{k_0 h}{2}\sin\theta\right)}{\frac{k_0 h}{2}\sin\theta}\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\frac{k_0 W}{2}\cos\theta}\right\}$$




(a) $E$-plane (b) $H$-plane

Typical $E$- and $H$-plane patterns of each microstrip patch